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PROPAGATION OF AN ARBITRARILY ORIENTED RECTILINEAR CRACK DURING EXTENSION OF A PLATE* **/48

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The effect of the orientation of a rectilinear crack, on its propagation during uniaxial extension of a plate is mathematically analyzed, assuming that the material of the isotropic plate obeys Hooke's law and the external force acts perpendicularly to the crack. The initial direction of propagation of the crack is always close to the direction normal to the line of action of the external force. A formula for calculating the magnitude of the external force at incipient propagation of the crack and the critical breaking stress is derived. At large angles made by the plane of the crack and the tangent to its initial propagation path, the crack propagated at stresses below unity.

The problem of the propagation of a rectilinear crack in ~~a plate~~ subjected to tensile stresses by external forces symmetric with respect to the plane of the crack has been considered previously (Bibl.1, 2, 4, 7, 8). However, the effect of the orientation of the crack on its propagation during extension of the plate has not been sufficiently studied. In this paper we give certain results of analytic and experimental studies of the propagation of a rectilinear crack of arbitrary orientation under uniaxial extension of the plate.

The analytic solution of the problem is based on other papers (Bibl.1, 3) and on certain assumptions as to the initial direction of propagation of the crack.

Section 1. Stresses Near the Ends of a Crack in an Elastic Plate

Let us consider an unbounded isotropic plate with a rectilinear crack (or notch) of length 2ℓ. We introduce a system of rectangular Cartesian coordinates xOy (Fig.1), and will consider that the material of the plate obeys Hooke's law until failure (the thickness of the plate is taken as unity).

At infinitely distant points of the plate, let tensile forces of an intensity p directed at an angle α to the line of the crack (Fig.1), act on the plate.

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* Some of the results of this work have been reported by V.V.Panasyuk and L.T.Berezhnitskiy at the II All-Union Conference on Theoretical and Applied Mechanics.

** Numbers in the margin indicate pagination in the original foreign text.

Let us determine, for this problem, the components of the stress tensor near the ends of the crack.

The components σ_x, σ_y, σ_{xy} of the stress tensor at an arbitrary point of an

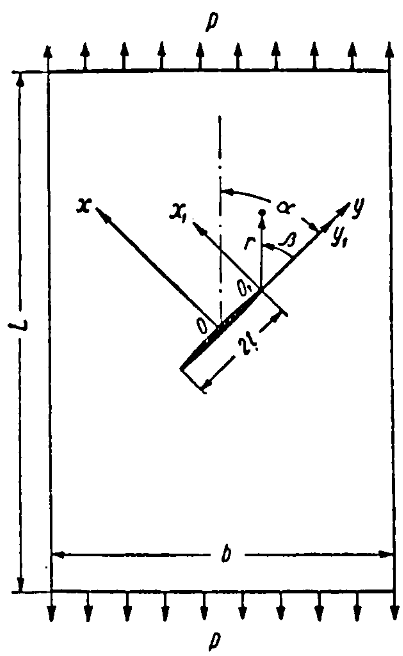

Fig.1

elastic plate are expressed in terms of the analytic functions $\Phi(z)$ and $\Omega(z)$ by the aid of the relations (Bibl.3):

$$\sigma_x + \sigma_y = 2\,[\Phi(z) + \overline{\Phi(z)}];$$
$$\sigma_y - \sigma_x + 2i\sigma_{xy} = 2\,[(\bar{z} - z)\,\Phi'(z) + \overline{\Omega}(z) - \Phi(z)], \qquad (1)$$

where $z = x + iy$, $\bar{z} = x - iy$.

In the case under consideration, the functions $\Phi(z)$ and $\Omega(z)$ are of the form (Bibl.3)

$$\Phi(z) = \frac{(2\Gamma + \overline{\Gamma}')\,z}{2\sqrt{z^2 - l^2}} - \frac{1}{2}\overline{\Gamma}';$$
$$\Omega(z) = \frac{(2\Gamma + \overline{\Gamma}')z}{2\sqrt{z^2 - l^2}} + \frac{1}{2}\overline{\Gamma}', \qquad (2)$$

where $\Gamma = \frac{p}{4}$, $\Gamma' = \frac{p}{2} - e^{-2i\alpha}$.

The square root represents the branch for which $z^{-1}\sqrt{z^2 - \ell^2} = 1$ as $z \to \infty$. /49

To study the stress state near the ends of the crack, it is of advantage to pass from the coordinates xOy to the new coordinates $x_1 O_1 y_1$, and also to use the polar coordinates (r, β) as shown in Fig.1. Then, making use of the formula of transition to a new coordinate system ($z_0 = \ell$)

$$\begin{aligned} \Phi_1(z_1) &= \Phi(z_1 + z_0); \\ \Omega_1(z_1) &= \Omega(z_1 + z_0) + (\bar{z}_0 - z_0)\overline{\Phi}'(z_1 + z_0), \end{aligned} \tag{3}$$

the components of the stress tensor, in the polar coordinates r, β with the center at the vertex of the crack, can be written, on the basis of eqs.(1), as follows:

$$\begin{aligned} \sigma_r + \sigma_\beta &= 4\,\mathrm{Re}\,\Phi_1(z_1); \\ \sigma_\beta - \sigma_r + 2i\sigma_{r\beta} &= 2[(\bar{z}_1 - z_1)\,\Phi_1'(z_1) + \\ &\quad \overline{\Omega}_1(z_1) - \Phi_1(z_1)]e^{2i\beta}. \end{aligned} \tag{4}$$

Equations (2) and (3) will yield

$$\begin{aligned} \Phi_1(z_1) &= \frac{p}{4}(1 - e^{2i\alpha})\frac{z_1 + l}{\sqrt{2z_1 l + z_1^2}} + \frac{p}{4}e^{2i\alpha}; \\ \Omega_1(z_1) &= \frac{p}{4}(1 - e^{2i\alpha})\frac{z_1 + l}{\sqrt{2z_1 l + z_1^2}} - \frac{p}{4}e^{2i\alpha}. \end{aligned} \tag{5}$$

Making further use of eqs.(4) and (5), we find

$$\begin{aligned} \sigma_r + \sigma_\beta &= p\,\mathrm{Re}\left[(1 - e^{2i\alpha})\frac{z_1 + l}{\sqrt{2z_1 l + z_1^2}}\right] + p\cos 2\alpha; \\ \sigma_\beta - \sigma_r + 2i\sigma_{r\beta} &= p\left[\frac{(1 - e^{2i\alpha})}{2}\,\frac{(z_1 - \bar{z}_1)l^2}{\sqrt{(2z_1 l + z_1^2)^3}} + \right. \\ &\quad \left. + i\sin 2\alpha\frac{z_1 + l}{\sqrt{2z_1 l + z_1^2}} - \cos 2\alpha\right]e^{2i\beta}. \end{aligned} \tag{6}$$

Consider now a small region near the ends of the crack, i.e., the point set for which the inequality $|z_1| \ll \ell$ holds. In the case $|z_1| \ll \ell$ we may write

$$\frac{z_1 + l}{\sqrt{2z_1 l + z_1^2}} = \frac{z_1 + l}{\sqrt{2z_1 l}}\left(1 + \frac{z_1}{2l}\right)^{-\frac{1}{2}} = \frac{1}{\sqrt{2}}\left(\frac{l}{z_1}\right)^{\frac{1}{2}} + \frac{3}{4\sqrt{2}}\left(\frac{z_1}{l}\right)^{\frac{1}{2}} - \frac{5}{32\sqrt{2}}\left(\frac{z_1}{l}\right)^{\frac{3}{2}} + \cdots \tag{7}$$

Writing similar representations for each term of eqs.(6) and combining the terms with the same powers of $\left(\frac{r}{l}\right)$, ($z_1 = r\,e^{i\beta}$, $r \ll l$), we obtain the following /50 expressions for the components of the stress tensor:

$$\sigma_r = \frac{p}{4}\sqrt{\frac{l}{2r}}\left[\sin^2\alpha\left(5\cos\frac{\beta}{2} - \cos\frac{3\beta}{2}\right) + \sin\alpha\cos\alpha\left(-5\sin\frac{\beta}{2} + \right.\right.$$
$$\left.\left. + \sin\frac{3\beta}{2}\right)\right] + p\cos 2\alpha\cos^2\beta + 0\left(\sqrt{\frac{r}{l}}\right);$$
$$\sigma_\beta = \frac{p}{4}\sqrt{\frac{l}{2r}}\left[\sin^2\alpha\left(3\cos\frac{\beta}{2} + \cos\frac{3\beta}{2}\right) - 3\sin\alpha\cos\alpha \times\right.$$
$$\left.\times\left(\sin\frac{\beta}{2} + \sin\frac{3\beta}{2}\right)\right] + p\cos 2\alpha\sin^2\beta + 0\left(\sqrt{\frac{r}{l}}\right); \tag{8}$$
$$\sigma_{r\beta} = \frac{p}{4}\sqrt{\frac{l}{2r}}\left[\sin^2\alpha\left(\sin\frac{\beta}{2} + \sin\frac{3\beta}{2}\right) + \sin\alpha\cos\alpha \times\right.$$
$$\left.\times\left(\cos\frac{\beta}{2} + 3\cos\frac{3\beta}{2}\right)\right] - \frac{1}{2}p\cos 2\alpha\sin 2\beta + 0\left(\sqrt{\frac{r}{l}}\right),$$

where the term $0\left(\sqrt{\frac{r}{l}}\right)$ indicates that the residual term of the series is of the order $\left(\frac{r}{l}\right)^{\frac{1}{2}}$.

We note that, for the general case of a two-dimensional extension of a plate with a rectilinear crack, the formulas for the components of the stress tensor will have an analogous form, as shown elsewhere (Bibl.6, 9).

Section 2. Determination of the Value of the Critical External Stresses

It has been shown (Bibl.1) that external forces applied to a body with a macroscopic crack will become critical if the resultant tensile stresses in the neighborhood of the ends of the crack, calculated by the methods of classical elasticity theory (without taking account of the cohesive forces between the edges of the crack) increase by the law

$$\frac{K}{\pi\sqrt{s}} \quad \text{at} \quad s \to 0, \tag{9}$$

where s is a small distance from the vertex of the crack, and K is a material constant; in the case of plane strain, we have

$$K = \sqrt{\frac{\pi E\gamma}{1-\nu^2}}; \tag{10}$$

and, in the case of plane stress,

$$K = \sqrt{\pi E\gamma}. \tag{11}$$

In eqs.(10) and (11), E is Young's modulus, ν is Poisson's ratio, and γ is the intensity of surface energy.

We will further consider (Bibl.5) that the initial propagation of the crack takes place along lines normal to which the tensile stresses reach the maximum possible intensity. On the basis of this assumption, and also in accordance with the law (9), we obtain the following equation for determining the critical values of the external forces $p = p_*$: /51

$$\lim_{r\to 0} \sqrt{r}\,\sigma_\beta\left(\frac{r}{l}, \alpha, \beta_*\right) = \frac{K}{\pi}, \tag{12}$$

where the angle β_* which determines the initial direction of propagation of the crack, satisfies the relation

$$\lim_{r\to 0}\left\{\sqrt{r}\left[\frac{\partial\sigma_\beta\left(\frac{r}{l}, \alpha, \beta\right)}{\partial\beta}\right]_{\beta=\beta_*}\right\} = 0. \tag{13}$$

Making use of eqs.(8) it is possible, after the necessary transformations, to write eq.(13) as follows:

$$\sin^2\alpha\left(\sin\frac{\beta_*}{2} + \sin\frac{3\beta_*}{2}\right) + \sin\alpha\cos\alpha\left(\cos\frac{\beta_*}{2} + 3\cos\frac{3\beta_*}{2}\right) = 0. \tag{14}$$

Solving eq.(14) for β_*, it is easy to find those values of the angle β_* at which σ_β reaches its maximum intensity. For $0 \le \alpha \le \pi/2$, such values of the angle β_* are determined by the formula

$$\beta_* = -2\arcsin\sqrt{\frac{6\cot^2\alpha + 1 - \sqrt{8\cot^2\alpha + 1}}{2(9\cot^2\alpha + 1)}}. \tag{15}$$

Substituting the value of the stress σ_β represented by eq.(8) into eq.(12) and passing to the limit as $r \to 0$, will yield the following formula for determining the limiting value of the external stresses $p = p_*$:

$$p_* = \frac{K\sqrt{2}}{\pi\sqrt{l}}\,\frac{1}{\cos^2\frac{\beta_*}{2}\left(\sin^2\alpha\cos\frac{\beta_*}{2} - 3\sin\alpha\cos\alpha\sin\frac{\beta_*}{2}\right)}, \tag{16}$$

where the angle β_* is determined by eq.(15).

On the basis of eqs.(15), (16), and (10), for $\alpha = \pi/2$, we find

$$\beta_* = 0, \qquad p_* = \frac{K\sqrt{2}}{\pi\sqrt{l}} = \sqrt{\frac{2E\gamma}{\pi(1-\nu^2)l}},$$

i.e., we obtain the well-known Griffith formula for a plate with a rectilinear crack when the external stresses p act in a direction perpendicular to the line of the crack.

A graph for the variation of the initial direction of propagation of the crack (value of angle β_*) with orientation of the crack (angle α) is given in Fig.2, indicated, in accordance with eq.(15), by the heavy line. It will be clear from the diagram that the initial direction of propagation of the crack is always close to the direction perpendicular to the line of action of the external tensile stresses p. Figure 3 is a graph of the correlation between the quantity $p_* \frac{\pi\sqrt{l}}{K\sqrt{2}}$ with the angle α, plotted according to eq.(16).

Section 3. Experimental Studies

To verify our theoretical studies we ran experiments (see Fig.1) on plates of sheet glass (silicate and organic) whose dimensions $L \times b \times h$ are given in /52

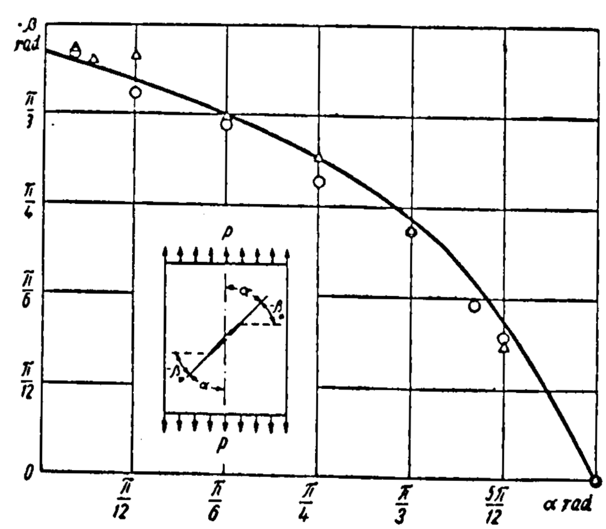

Fig.2

Tables 1 and 2. A hole 6 mm in diameter was drilled in the center of each plate. By the aid of a glass cutter, a rectilinear crack was made in each plate, in the direction of the hole diameter, at a certain angle α (Fig.1) to the longitudinal axis of the plate. Then, by a special device (application of

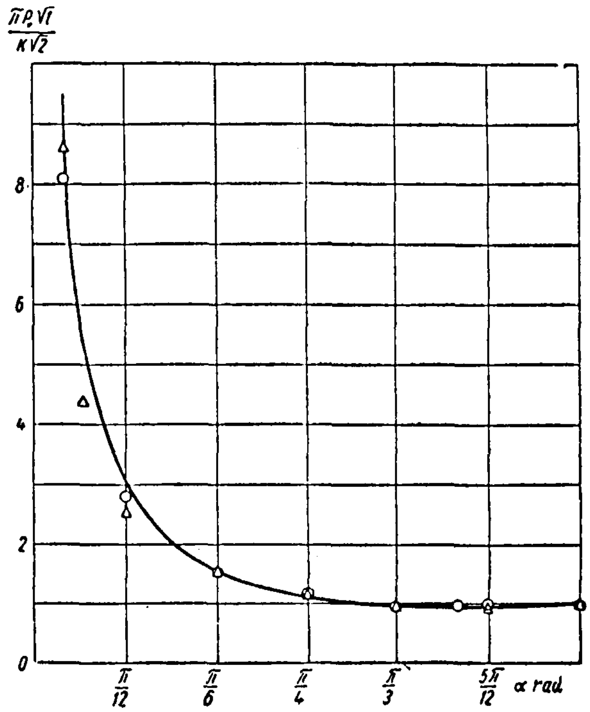

Fig.3

forces to the edges of the hole in a direction perpendicular to the line of the crack), the initial crack was extended through the entire thickness of the plate and was propagated on both sides to a certain length $2l$. We measured the length $2l$ of the initial crack and the angle α. In this way we prepared plates of silicate and organic glass with cracks of varying length ($2l_\alpha$) and varying angles α at $0 < \alpha \leq \pi/2$. /53

The plates were checked on the polaroscope to establish the nature of their residual stress. The specimens so prepared (plates) were subjected to tension in the direction of the longitudinal axis of the plate on an MR-0.5 tensile testing machine with a speed of 6.6×10^{-6} m/sec for the active grip. The specimens were connected with the grips of the machine in two ways: 1) by drilling holes in the specimens far from the crack and attaching the plate by bolts to the metal grips, and 2) by cementing dermatin (leatherette) clamps to the end of the plate. The two forms of attachment gave the same results in the experiments (within the accuracy of measurement).

During extension of a plate with a crack directed at an angle α to the line of extension of the plate, the external force $p = p_*$ at which the crack began to propagate and the plate ruptured was determined. From the experimental data we determined the critical (breaking) stress by the formula /54

$$p_{*\alpha} = \frac{P_{*\alpha}}{bh}\left(\frac{\text{dan}}{\text{mm}^2}\right),$$

where the subscript α means that the quantities in question refer to a crack making an angle α with the longitudinal axis of the plate.

TABLE 1

L_{mm}	b_{mm}	h_{mm}	l_{mm}	α rad	P_* dan	$\beta_*^{(l)}$ rad	$\beta_*^{(r)}$ rad	$\beta_*^{(tot)}$ rad	$p_*\sqrt{l}$	η
250	249	2.20	44.00	$\pi/2$	132.5	0	0	0	1.61	1.01
238	179	1.73	35.50	$\pi/2$	81.4	0	0	0	1.57	0.99
238	178	1.73	34.00	$\pi/2$	84.8	0	0	0	1.61	1.01
238	178	1.69	33.35	$5\pi/12$	83.8	0.4724	0.4918	0.4821	1.61	1.01
238	178	1.69	30.95	$5\pi/12$	85.3	0.4829	0.3569	0.4199	1.58	0.99
250	150	2.09	32.47	$5\pi/12$	69.8	0.4390	0.3892	0.4141	1.53	0.96
250	150	2.09	32.54	$5\pi/12$	84.8	0.3629	0.3473	0.3551	1.54	0.97
250	150	2.10	33.03	$7\pi/18$	77.5	0.4236	0.4599	0.4467	1.43	0.90
250	150	2.11	35.91	$7\pi/18$	90.7	0.5973	0.5229	0.5601	1.72	1.08
238	178	1.70	34.85	$\pi/3$	91.2	0.7549	0.7139	0.7344	1.76	1.10
238	178	1.79	31.47	$\pi/3$	83.8	0.6062	0.7186	0.6624	1.48	0.92
250	150	2.03	36.32	$\pi/3$	71.2	0.7230	0.7596	0.7413	1.41	0.88
238	178	1.79	34.88	$\pi/4$	95.7	0.7878	0.8526	0.8202	1.78	1.12
238	178	1.92	37.32	$\pi/4$	112.8	0.9393	0.9251	0.9322	2.01	1.26
238	178	1.80	36.00	$\pi/4$	110.4	0.8304	0.8090	0.8197	2.07	1.30
250	179	2.02	56.53	$\pi/6$	114.7	0.9326	1.0641	0.9984	2.39	1.50
250	180	2.22	55.57	$\pi/6$	144.2	1.1188	1.0780	1.0984	2.69	1.69
250	180	2.16	80.16	$\pi/6$	98.1	0.9216	1.0013	0.9615	2.27	1.42
300	180	2.22	92.68	$\pi/12$	224.5	1.0653	1.0177	1.0415	5.42	3.40
300	124	1.75	44.18	$\pi/12$	213.0	1.1688	1.0728	1.1208	3.96	2.48
300	121	1.91	82.67	$\pi/12$	103.0	1.1950	1.1234	1.1592	4.05	2.54
300	120	1.88	117.67	$\pi/36$	270.0	1.2273	1.2256	1.2265	12.95	8.12

TABLE 2

L_{mm}	b_{mm}	h_{mm}	l_{mm}	α rad	P_* dan	$\beta_*^{(l)}$ rad	$\beta_*^{(r)}$ rad	$\beta_*^{(tot)}$ rad	$p_*\sqrt{l}$	η
390	250	1.02	53.00	$\pi/2$	114.9	0	0	0	3.28	1.00
390	250	1.13	42.06	$5\pi/12$	131.0	0.3982	0.3912	0.3947	3.00	0.92
390	250	1.11	43.88	$\pi/3$	129.0	0.6612	0.7794	0.7203	3.08	0.94
440	250	1.13	41.24	$\pi/4$	168.0	0.9440	0.9160	0.9300	3.82	1.16
370	250	1.00	45.15	$\pi/6$	183.5	1.0132	1.0696	1.0414	4.93	1.51
480	250	1.08	62.88	$\pi/12$	285.0	1.2130	1.2263	1.2197	8.37	2.55
340	161	1.04	120.85	$2\pi/45$	219.5	1.2200	1.1917	1.2058	14.45	4.41
520	173	1.02	138.83	$\pi/36$	423.0	1.1933	1.2866	1.2399	28.30	8.63

It must be noted that on silicate glass specimens we observed the propagation (rotation) of the initial crack at stresses p somewhat lower than p_* (at p of about 0.95 p_*). This phenomenon was observed only at large values of the angle α.

After rupture of the plate, the initial length of the crack $2l$ was refined by measuring its value on the horizontal comparator IZA-2. From the data of the experimental measurements for each plate we calculated the value of $p_{*\alpha}\sqrt{l_\alpha}$ and the ratio

$$\eta(\alpha) = \frac{p_{*\alpha}\sqrt{l_\alpha}}{p_{*\frac{\pi}{2}}\sqrt{l_{\frac{\pi}{2}}}} = \frac{\pi p_{*\alpha}\sqrt{l_\alpha}}{K\sqrt{2}}.$$

Columns 10 and 11 of Tables 1 and 2 give the results of these calculations for silicate and organic glass, respectively. The mean value of $\eta(\alpha)$ for each group of plates with the same angle α (see Tables 1 and 2) are plotted in Fig.3,

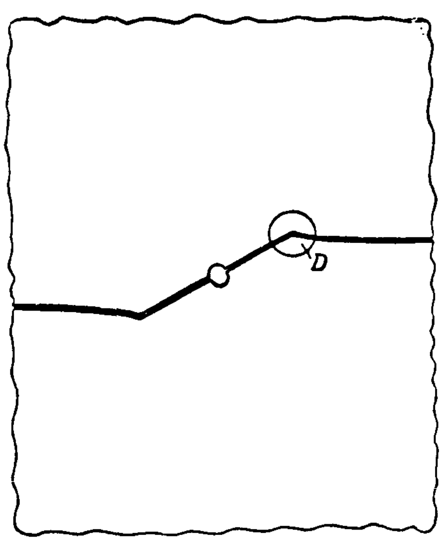

Fig.4

where the triangles refer to the organic-glass plates and the circles to the silicate-glass plates. The heavy line in this diagram represents the function $\eta(\alpha)$ plotted on the basis of eq.(16).

For each plate we also measured the angles β_* of the initial propagation (rotation) of the crack. For this purpose, we used the large BMI-1 instrument microscope which, in combination with a goniometric eyepiece and a projection device, made it possible to do this with considerable accuracy. In Fig.1, the quantity β_* is the angle made by the plane of the crack and the tangent to the path of initial propagation of that crack. In the experimental determination of the angle β_*, we assumed that the tangent to the path of initial propagation of the crack was a straight line passing through the vertex of the crack and a point located on the path of propagation of the crack at a distance of 0.3 mm. To illustrate the process of measuring the angle β_*, Fig.4 gives a photograph of an organic-glass plate for the crack at $\alpha = \pi/3$ after rupture. Figure 5 is an image of the region D of Fig.4 at a magnification of 30 ×, as observed in the projection device of the microscope. This picture shows a small part of the initial crack 2ℓ, part of the path of the crack at rupture, the point A at a distance of 0.3 mm from the vertex of the crack, and the measured angle β_*. The distance 0.3 mm was adopted in accordance with the scale divisions of the ocular head of the microscope.

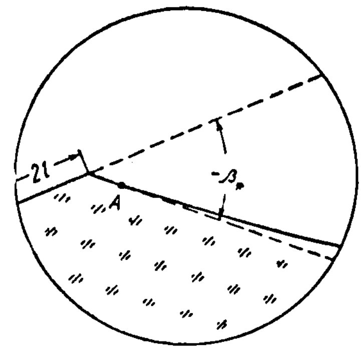

Fig.5

For all plates used in the experiment we measured the angle β_* for both ends of the crack, i.e., $\beta_*^{(left)}$ and $\beta_*^{(right)}$.

The results of the measurements are given in columns 7, 8, and 9 of Tables 1 and 2, where $\beta_*^{tot} = 1/2\ (\beta_*^{(left)} + \beta_*^{(right)})$. The mean values of the angles β_*^{tot} for each group of plates (silicate glass, Table 1 and organic glass, Table 2), are plotted against the angle α, indicated by the circles and tri- /55 angles of Fig.2. It will be clear from Figs.2 and 3 that the experimental data confirm the results of our theoretical calculations.

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